

Algèbre linéaire: ACP, schéma

Cours 1^{ière} année

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trschema_acp_ps.ps

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INP-ENSAT

Données

- Données

$$X = (x_{ij})_{ij} \quad i = 1, \dots, n \quad j = 1, \dots, p$$

- Moyennes

$$\bar{x}_{.j} = \frac{1}{n} \sum_i x_{ij}$$

- Variances

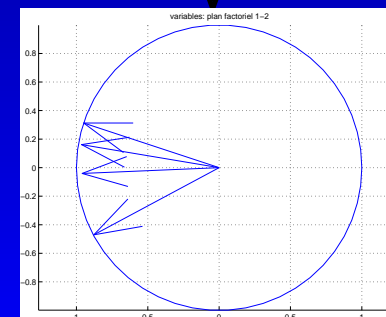
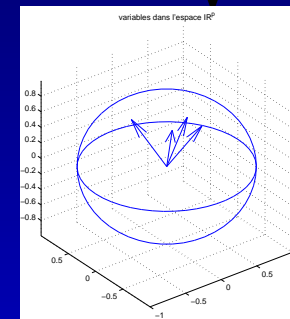
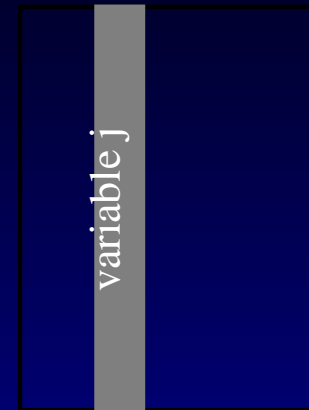
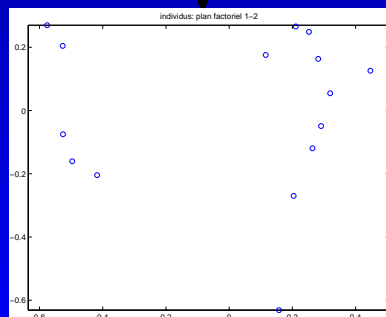
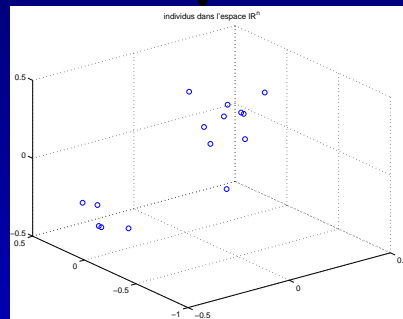
$$s_j^2 = \frac{1}{n} \sum_i (x_{ij} - \bar{x}_{.j})^2$$

$$y_{ij} = \frac{x_{ij} - \bar{x}_{.j}}{s_j}$$

- Données centrées normées

$$z_{ij} = \frac{x_{ij} - \bar{x}_{.j}}{\sqrt{n} s_j}$$

Schéma ACP



ACP Normée

- $Z = V\Sigma^tU$
- $\psi = ZU$
- $\psi^Y = YU = \sqrt{n}\psi$
- $\phi = {}^tZV$

(i) $v_{.j} = \frac{1}{\sqrt{\lambda_j}} Z u_{.j}$

(ii) $u_{.j} = \frac{1}{\sqrt{\lambda_j}} {}^t Z v_{.j}$

(iii) $\phi_{.j} = \sqrt{\lambda_j} u_{.j}$

(iv) $\psi_{.j} = \sqrt{\lambda_j} v_{.j}$

(v) $\|\psi_{.j}\|^2 = \lambda_j = \text{inertie de } \psi_{.j}^Y$

(vi) $\|\phi_{.j}\|^2 = \lambda_j$